AN INVESTIGATION FOR REAL TIME COMPREHENSIVE SENSOR MODELING

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Abstract
Several commercial high resolution imagery satellites, were successfully launched and several commercial government missions in different countries such as US, Russia, France and India scheduled launches of these satellites. Successful exploitation of the high accuracy potential of these systems depends on good mathematical models for the sensor modeling. For real-time applications, simpler image geometry models are often used, requiring much shorter computation times. Depending upon accuracy requirement and image exploitation applications, the rational functions, Direct Linear Transformation (DLT), Self calibration DLT (SDLT), 2D projective, polynomials, 3D affine, multiquadrics and thin plate spline approximate models may be used for real-time positioning. This paper discusses seven non-rigorous sensor models suitable for real-time use, with emphasis on a comprehensive sensor model. The flexibility and good accuracy of the non-rigorous models with a number of US (Ikonos), India (IRS-1C), Russia (KFA-1000) and French (SPOT 4) sensors with resolution varying from 0.82m to 10 m are demonstrated. The test areas cover parts of north-east, west and south of Iran. Different methods for determining ground control points (GCPs) and independent check points (ICPs) including GPS, digital maps and models after absolute orientation were used.

1. Introduction
An image geometry model is needed to determine the correct ground position of a point visible on an image of the earth. An image geometry model is alternately called an image sensor model, sensor model, or image mathematical model. Successful exploitation of the high accuracy potential of high resolution satellite imagery (HRSI) such as Ikonos-2, EROS-A1, QuickBird-2 and SPOT-5 depends on a comprehensive mathematical modeling of the imaging sensor. The geometry of most HRSI is based on the pushbroom principle. An orbital parameter model can be applied to stereo space imagery in order to determine exterior orientation parameters, and a number of papers have been published on different approaches for the position and attitude determination of SPOT, IRS-1C/D, MOLS and other sensors (Gugan, 1986; Dowman, 1991; Fraser & Shao, 1996; Radhadevi et al., 1998). For example, bundle adjustment methods for SPOT Level 1A and 1B stereo pairs (Valadan Zoej & Petrie, 1998), MOMS-O2 stereo images (Valadan Zoej, 1997) and IRS-1C stereo pairs (Valadan Zoej & Foomani, 1999) have been developed and tested. Unfortunately most high resolution satellite vendors do not intend to publish their sensor models and ephemeris data. For example at this writing, Space Imaging has refused to release information on the camera model for Ikonos, as well as data on the precise in-flight position and attitude of the imaging sensor. This means that a large number of photogrammetric parameters are both unknown, and not readily determinable from the imagery alone. The very long focal length (10 m) and narrow angle of view (0.93°) and swath (~11 km) will likely make an orbital resection unstable, and even if many GCPs and several images are used, an accurate solution might not be possible. Therefore an end user needs a replacement sensor model for photogrammetric processing.

Recently, several 2D and 3D approaches have been reported to tackle this issue (Fraser et al., 2002; Valadan et al., 2002; Sadeghian, 2002; Hanley & Fraser, 2001; Sadeghian et al., 2001a, 2001b, 2002). A possible solution is to use approximate models. Non-rigorous models are mathematical models, rather than physical models. The physical model is blindly expressed in mathematical terms. The parameters and coefficient of these approximate sensor models can be computed from a rigorous sensor model by an analytical triangulation adjustment. This approach call terrain-independent. Without knowing the physical sensor model, GCPs have to be collected to solve the coefficient. This approach is called terrain-dependent. Terrain-dependent method does not require interior orientation parameters or orbit ephemeris information.
The image to object space transformation solution is based only upon ground control points (GCPs). This is an advantage for processing the new HRSI.

Among approximate models, the Rational Function Model (RFM) seems to gain popularity and Space Imaging Inc. has adopted the RFM as an alternative sensor model for image exploitation. There are few publications that address background, advantages, disadvantages, accuracy and stability of RFM (Tao & Hu, 2001; Grodecki & Dail, 2001; Dowman & Dolloff, 2000; Yang, 2000; Madani, 1999; Greve et al., 1992). In this paper the possibility of using non-rigorous models for 2D ground point determination were explored and investigated.

2. Ikonos in Comparison to IRS-1C, KFA-1000 and SPOT4
Ikonos-2 was successfully launched on September 24, 1999 into a polar sun-synchronous orbit with an inclination of 98.1° at a height of 680 km. The satellite payload consists of both 1m panchromatic and 4-band (blue, green, red, near-infrared), 4m multispectral 11-bit sensors. Ikonos travels at a speed of 7 km/s, which allows it to orbit the planet every 98 minutes. Details of the Ikonos satellite and its camera systems are given by Gerlach (2000) and Grodecki & Dail (2001). A comparison of technical data for Ikonos, IRS-1C, KFA-1000 and SPOT4 sensors is shown in Table I.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Spatial resolution</th>
<th>Resolution</th>
<th>Spectral</th>
<th>Radiometric</th>
<th>Temporal</th>
<th>Altitude (km)</th>
<th>Pixel Size (µm)</th>
<th>No. of pixels</th>
<th>Swath width (km)</th>
<th>Stereo principle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ikonos</td>
<td>0.8</td>
<td>Panch</td>
<td>Vis</td>
<td>1</td>
<td>11</td>
<td>680</td>
<td>12</td>
<td>11</td>
<td>13500</td>
<td>Flexible</td>
</tr>
<tr>
<td>IRS-1C</td>
<td>5.8</td>
<td>Panch</td>
<td>Vis</td>
<td>2</td>
<td>6</td>
<td>817</td>
<td>7</td>
<td>3*4096</td>
<td>70</td>
<td>Cross track</td>
</tr>
<tr>
<td>KFA-1000</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>270</td>
<td>60/1 mm</td>
<td>-</td>
<td>30*30 cm</td>
<td>Along track</td>
</tr>
<tr>
<td>SPOT4</td>
<td>10.0</td>
<td>Panch</td>
<td>Vis</td>
<td>2</td>
<td>8</td>
<td>830</td>
<td>13</td>
<td>6000</td>
<td>60</td>
<td>Cross track</td>
</tr>
</tbody>
</table>

As indicated in Table 1, there is a linear relationship between spatial resolution and swath width. Thus, Ikonos imagery, while having the advantage of 1m resolution, has a distinct disadvantage compared to IRS-1C/1D, KFA-1000 and SPOT in terms of area coverage, since this is achieved at the cost of additional time and expense, both in acquiring and processing the imagery.

The Ikonos system is based on a pushbroom scanner with a lens of 10 m focal length that has been folded into a 2m length through the use of two additional flat mirrors incorporated into its telescope. This significantly reduces the physical size and weight of the camera. The CCD array in the panchromatic sensor has 13500 x 12 x 12 µm pixels. The IRS-1C/D, on the other hand, has a 1m focal length lens and the panchromatic camera comprises three CCD arrays each having 4096 active sensor elements with pixel sizes of 7 x 7 µm pixels. The three CCD lines are not parallel to each other and may not be orthogonal against the optical axis (Jacobsen, 1998). The optical design features an off-axis primary hyperboloid mirror, a spherical secondary mirror and an off-axis ellipsoidal tertiary mirror. Details of IRS-1C satellite and its sensor systems are given by Kasturirangan et al. (1996) and Joseph et al. (1996).

The KFA-1000 camera system is originally planned for interpretation purposes. Much interest has arisen for its possible use in medium-scale topographic mapping because of a good resolving power of the system. The adjacent photo strips had been exposed simultaneously with the KFA-1000 double camera system where the rotational angle between the camera unit is 16 degrees. The KFA-1000 photo has 5 fiducial marks, 4 in the center of each side and 1 in the photo center, So, the transformation to the calibrated fiducial mark coordinates is not a problem. The fiducials are superimposed onto the film and if there is sufficient contrast, observation of them can be difficult. The KFA-1000 imaging system has the advantage of being an optical frame sensor and is not made of a linear array sensor such as IKONOS, IRS-1C/D and SPOT. They do not have problems like shifts or variations between successive sensor orientations. However, they have problems like photographic processing and storage. The distortion of the optics of the KFA-1000 cannot be described with a radial-symmetrical model within a proper accuracy range, e.g. better than 3 µm. The photos of the KFA-1000 are 30 cm by 30 cm. Thus they cannot be scanned by standard photogrammetric scanners which allow formats up to 25 cm by 25 cm. The vertical accuracy on the other side is limited by mapping with space photos. The KFA-1000 has not been designed for optimal height accuracy. B/H for SPOT4, IRS-1C/1D and Ikonos is better but the difference in time between the recording of the same area by SPOT4 and IRS-1C/1D can cause some problems. For example, the reflectance of the ground may be changed. For KFA-1000 and Ikonos images such a problem with the stereo effect do not exist.
3. Real Time Sensor Models

The real-time comprehensive sensor model, must have some advantages including (Madani, 1999):

- Sensor independence
- Sufficient speed for real time implementation
- Support of any coordinate system
- Easy inclusion of external sensors into work flows via
- Image object grids used to generate upward projection
- High accuracy
- Ability to handle all types of imagery

Fast computation is useful for rectification, orthorectification, perspective scene generation, stereo reconstruction, DEM generation, feature editing, quality checking, etc.

In the following discussion, rational functions and their derivatives, as well as radial basis functions, are evaluated as potential approximate sensor models to substitute for the rigorous physical sensor model.

Rational Functions

The concept of the rational function model (RFM) was developed by Gyer, and has been used extensively by the US Defense Mapping Agency in their Digital Production System, PEGASUS for example (Greve, 1992). The RFM is also widely used by the intelligence community. Under the model, an image coordinate is determined from a ratio of two polynomial functions, in which the image (x, y) and ground coordinates (X, Y, Z) have all been normalized (OGC, 1999).

\[
x = \frac{P_1(X, Y, Z)}{P_2(X, Y, Z)} = \sum_{i=0}^{m_1} \sum_{j=0}^{m_2} \sum_{k=0}^{m_3} a_{ijk} X^i Y^j Z^k \sum_{j=0}^{n_2} \sum_{k=0}^{n_3} b_{ijk} X^i Y^j Z^k
\]

\[
y = \frac{P_3(X, Y, Z)}{P_4(X, Y, Z)} = \sum_{i=0}^{n_1} \sum_{j=0}^{n_2} \sum_{k=0}^{n_3} c_{ijk} X^i Y^j Z^k \sum_{j=0}^{n_2} \sum_{k=0}^{n_3} d_{ijk} X^i Y^j Z^k
\]

The RFM maps three-dimensional ground coordinates to image space for all types of sensors, such as frame, pushbroom, whiskbroom and SAR systems. Dowman & Dolloff (2000) summarise the advantages of rational functions as follows:

i) Universality: The method can be used to transform image coordinates from an image produced by any imaging sensor into object space coordinates; coefficients can implicitly contain all kinds of effects (sensor geometry, Earth curvature, refraction and self calibration parameters); partial images can be processed; and the user need only obtain and maintain one sensor model that covers many different sensors.

ii) Confidentiality: This allows image data to be used to create object coordinates without explicit knowledge of the sensor model; the military already make use of this and data from the new high resolution satellites will probably be processed using Universal Sensor Models.

iii) Efficiency: When used in the real-time loop of photogrammetric stereoplotters, fewer operations are required.

iv) Transfer: RFM makes transfer of image orientation data very straightforward.

Due to these characteristics, rational functions have attracted considerable interest in photogrammetry and remote sensing communities and they have been proposed by OGC as an image transfer standard (OGC, 1999). Rational function coefficients (RFCs), also called Rational Polynomial Coefficients (RPCs), are provided by vendors as a component of the image metadata. For example, the IKONOS RPC model is expressed simply as a ratio of two cubic polynomials and it differs by no more than 0.04 pixel from the rigorous, physical sensor model, with the RMS error being below 0.01 pixel (Grodecki & Dial, 2001). The DLT, SDLT, 2D projective and 3D affine equations are specialised forms of the RFM, and we now consider these models.
Direct Linear Transformation

In Eq. 1, if $m_1=m_2=m_3=n_1=n_2=n_3=1$ when $i+j+k=0$, $a_{ijk}=b_{ijk}=c_{ijk}=d_{ijk}=0$ when $i+j+k=0$, $b_{ijk}=1$ when $i+j+k=0$, and $b_{ijk}=d_{ijk}$, we obtain the DLT in which 11 linear orientation parameters define the relationship between 2D image space and 3D object space:

$$
\begin{align*}
    x &= \frac{a_1 + a_2X + a_3Y + a_Z}{1 + c_1X + c_2Y + c_Z}, \\
    y &= \frac{b_1 + b_2X + b_3Y + b_Z}{1 + c_1X + c_2Y + c_Z}.
\end{align*}
$$

Self Calibration Direct Linear Transformation

Twelve linear orientation parameters define the relationship between 2D image and 3D object space:

$$
\begin{align*}
    x &= \frac{a_1 + a_2X + a_3Y + a_Z}{1 + c_1X + c_2Y + c_Z} + a_{xy}, \\
    y &= \frac{b_1 + b_2X + b_3Y + b_Z}{1 + c_1X + c_2Y + c_Z}.
\end{align*}
$$

2D Projective Equations

In Eq. 1, if $m_1=m_2=n_1=n_3=1$ when $m_3=n_3=0$ and $i+j+k=0$, $a_{ijk}=b_{ijk}=c_{ijk}=d_{ijk}=0$ when $i+j+k=0$ and $b_{ijk}=d_{ijk}$, we obtain the 2D projective model. Eight parameters define the relationship between the object and image planes:

$$
\begin{align*}
    x &= \frac{a_1 + a_2X}{1 + c_1X + c_2Y}, \\
    y &= \frac{b_1 + b_2X}{1 + c_1X + c_2Y}.
\end{align*}
$$

3D Affine Transformation

In Eq. 1, if $m_1=m_2=m_3=1$ when $m_1=m_2=n_3=0$ and $i+j+k=1$, $a_{ijk}=b_{ijk}=0$ when $i+j+k=0$ and $b_{ijk}=d_{ijk}=1$, we obtain the 3D affine transformation. Eight parameters define the relationship between the 2D object and 3D object spaces:

$$
\begin{align*}
    x &= a_1 + a_2X + a_3Y + a_Z, \\
    y &= b_1 + b_2X + b_3Y + b_Z.
\end{align*}
$$

In the above functions, $x, y$ are the image coordinates; $X, Y, Z$ are the ground coordinates; and $a_1, b_1, c_1, d_1$ are transformation parameters.

4. Test Areas and Data

Test Area and Data Materials of Ikonos Geo Image

The Ikonos Geo panchromatic image employed covered an 8 x 7 km area of central Hamedan city in the west of Iran (see Figure 1). It was acquired on 7 October 2000 with a 20.4º off-nadir angle and 47.4º sun elevation. Carterra Geo products are georectified, which means that they are rectified to an inflated ellipsoid and selected projection, in this case UTM on the WGS84 datum. No terrain-correction model is applied so these images are only rectified, as opposed to orthorectified. The stated accuracy of the Carterra Geo products is specified as 50m CE90 exclusive of terrain displacement (Grodecki & Dail, 2001). In this investigation, the elevation within the Ikonos test area ranged from 1700 m to 1900 m. The GCPs/ICPs for the tests were extracted from NCC-product digital maps which employed a UTM projection on the WGS84 datum. In this instance the mapping scale was 1:1000 (see Figure 2), with the compilation have been carried out using 1:4000 scale aerial photographs (see Figure 3). The selected GCPs/ICPs in the imagery were distinct features such as building (see Figure 4) and pools corners, and wall and roads crossings, etc.
The image coordinates of the GCPs/ICPs were monoscopically measured using the PCI EASI/PACE software system. These image measurements were then input into the least-squares adjustment computations, for the parameters of the rational functions, DLT, SDLT, 3D affine and projective transformation.

Test Area and Materials of IRS-1C Image
The left image of an IRS-1C stereo pairs in ‘superstructure’ format was used in the testing for this research. The image was recorded on 27 August 1998 with an off-nadir view angle of 11.5º. The scene covered an area of 26 km\(^2\) over the Mashhad test area in the north-eastern Iran. The city of Mashhad was located in the central and north part of the image and the south-west corner of the scene covered a hilly area. The radiometric quality of the IRS-1C image was poor, which affected the quality of control point measurements. Thus, a preprocessing stage that included image enhancement was carried out. This involved the separate treatment of individual patches before the x,y coordinate measurements of the GCPs and independent check points (ICPs) were made on the image. The GCPs/ICPs for these tests are extracted from NCC-produced 1:25000 scale digital 3D maps with WGS84 datum and U.T.M. map projection, compiled from 1:40000 scale aerial photographs. A total of 65 well distributed GCPs/ICPs were selected and measured. These points were observed monoscopically to sub-pixel accuracy, estimated to be better than 0.5 pixel, on the image using a PCI EASI/PACE image processing system. The object space accuracy of the GCPs/ICPs was estimated to be about 7.5 m.

Test Area and Materials of KFA-1000 Photo
The KFA-1000 photo used in the test, had been taken in 1990 from the south of Iran, covering a flat test area. The altitude was 276 km and the photo size on the ground was 80*80 km\(^2\). The focal length of the camera was 1009 mm, and the original photo scale was about 1:272000. In this test GCPs/ICPs have been
measured on the model at a scale of 1:40000 aerial photos in DSR14 analytical plotter after completion of inner, relative and absolute orientation. The accuracy of the GCPs/ICPs were estimated to be better than 1m. Details of the testfield and image mensuration are given by Sadeghian et al. (2001a).

Test Area and Materials of SPOT 4 Image
The SPOT 4 level 1A image that was used covers the Kermanshah area in the west of Iran. It was acquired at September 1999. Height range of the terrain is 1300 m to 3500 m. A GPS survey to provide the test field of ground control and check points was carried out by N.C.C. staff in 2001. The positions and heights of GCPs/ICPs were measured using differential GPS techniques with 10 cm accuracy. The position of the GCPs/ICPs on the image were measured monoscopically using the PCI EASI/PACE package.

5. Practical Evaluation
All image-to-object space transformation computations were carried out with software written by the author. Least squares determinations of the parameters of each orientation model were carried out using all available GCPs, namely 34 for the Ikonos image, 45 for the IRS-1C scene, 21 for the KFA-1000 photo and 12 for the SPOT4 image. The ground coordinates of ICPs were then determined utilizing the derived parameters, and the differences between the photogrammetrically determined and map-recorded ground positions then formed the basis of the accuracy assessment phase. Tables II, III, IV and V show summaries of the root mean square error (RMSE) obtained for the series of 2D object point determinations, for the Ikonos, IRS-1C, KFA-1000 and SPOT4 images, respectively. Where ∆xy is represented as the square root of sum of ∆x and ∆y squares. Of the specific model options considered, The DLT; SDLT; 2D projective; 3D affine methods; and rational functions with 14 (10 in the numerators x and y and 4 plus the constant 1 in the denominator), 17 and 20 terms.

<table>
<thead>
<tr>
<th>Method</th>
<th>Control Points (n=34)</th>
<th>Check Points (n=20)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>∆x (m)</td>
<td>∆y (m)</td>
</tr>
<tr>
<td>2D Projective (8 term)</td>
<td>1.72</td>
<td>4.10</td>
</tr>
<tr>
<td>3D Affine (8 term)</td>
<td>0.64</td>
<td>0.71</td>
</tr>
<tr>
<td>DLT (11 term)</td>
<td>0.60</td>
<td>0.68</td>
</tr>
<tr>
<td>SDLT (12 term)</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>Rational 14 term</td>
<td>0.59</td>
<td>0.58</td>
</tr>
<tr>
<td>Rational 17 term</td>
<td>0.59</td>
<td>0.54</td>
</tr>
<tr>
<td>Rational 20 term</td>
<td>0.55</td>
<td>0.53</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Method</th>
<th>Control Points (n=45)</th>
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</tr>
</thead>
<tbody>
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<td></td>
<td>∆x (m)</td>
<td>∆y (m)</td>
</tr>
<tr>
<td>2D Projective (8 term)</td>
<td>17.64</td>
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<td>3.62</td>
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<td>1.98</td>
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</tr>
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<td>1.18</td>
</tr>
<tr>
<td>Rational 14 term</td>
<td>0.93</td>
<td>0.77</td>
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<td>Rational 17 term</td>
<td>0.91</td>
<td>0.69</td>
</tr>
<tr>
<td>Rational 20 term</td>
<td>0.89</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Also, using a 44 term rational function, 25 in the numerator and 19 plus the constant 1 in the denominator, a number of additional tests were carried out for the image. The power of RFMs varied for each dimension as well as for the numerator and the denominator. The optimal RFM for Ikonos data turned out to have 17 terms as shown in table II. Some sensors would not need that many terms and the solution may well have highly correlated coefficients. Over-parameterization causes instability and indeterminability in the least squares solutions. The significant coefficients for a particular sensor can be selected by trial and error. However Tao and Hu (2001) used the regularization technique with some success to overcome this drawback.
## Table IV. $\Delta x$, $\Delta y$ RMSE values achieved in UTM coordinates of the KFA-1000 data over the Khozestan project area.

<table>
<thead>
<tr>
<th>Method</th>
<th>Control Points (n=21)</th>
<th>Check Points (n=7)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta x$ (m)</td>
<td>$\Delta y$ (m)</td>
</tr>
<tr>
<td>2D Projective (8 term)</td>
<td>0.06</td>
<td>0.08</td>
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<tr>
<td>3D Affine (8 term)</td>
<td>0.30</td>
<td>0.30</td>
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<tr>
<td>DLT (11 term)</td>
<td>0.05</td>
<td>0.05</td>
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<td>SDLT (12 term)</td>
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<td>0.03</td>
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<tr>
<td>Rational 14 term</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Rational 17 term</td>
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<td>0.03</td>
</tr>
<tr>
<td>Rational 20 term</td>
<td>0.03</td>
<td>0.03</td>
</tr>
</tbody>
</table>

## Conclusions and Discussions

In this paper the accuracy potential of Ikonos Geo, IRS-1C, KFA-1000 and SPOT4 images was investigated. This investigation has shown that Ikonos Geo imagery has high geometric integrity. When distinct object features such as building corners or roads crossings are used, an accuracy of 1 m can be readily achieved for Ikonos Geo with relatively straightforward linear models. We tested the possibility of using seven non-rigorous model for camera modeling of Ikonos, IRS-1C, KFA-1000 and SPOT4 image. Since the DLT method considers elevation also, the results are better than the 2D projective approach method. The question of whether one should employ an empirical sensor orientation model seems to have no simple answer. However it seems reasonable to conclude that the RFM can be used for real time camera modeling. In the DLT method, the whole linear array system is considered in exactly the same way as a photograph and the image coordinate system is coinciding with the pixel coordinate system. 11 linear orientation parameter must be determined for each image in this method since the parameter are linear, linearization of the DLT equations is simple and this will save in the time and power required for the bundle adjustment program. The flexibility and favorable accuracy of the empirical orientation model approach has been demonstrated with Ikonos Geo image and the method should be equally useful for other high resolution satellite imaging systems. Further research is needed regarding both the robustness, accuracy and error sources in RFM, and automatic methods of selecting an appropriate number of coefficients for HRSI.

## Acknowledgments

The author would like to acknowledge Iranian Remote Sensing Center (IRSC) for providing Ikonos and IRS-1C data, and National Cartographic Center (NCC) for providing SPOT image, GCPs/ICPs and 1:1000 scale digital maps of the Hamedan test field and National Geographic Organization (NGO) for providing KFA-1000 photo.

## References


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Table V. $\Delta x$, $\Delta y$ RMSE values achieved in UTM coordinates of the SPOT4 data over the Kermanshah.

<table>
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<td>$\Delta x$ (m)</td>
<td>$\Delta y$ (m)</td>
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<tr>
<td>3D Affine (8 term)</td>
<td>0.44</td>
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<td>DLT (11 term)</td>
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<td>SDLT (12 term)</td>
<td>1.77</td>
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<td>Rational 14 term</td>
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<td>Rational 17 term</td>
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